APM 664: Combinatorial Optimization
Winter 2018

- **Instructor:** Eddie Cheng, Office: 348 MSC Tel: 370-4024 email:echeng@oakland.edu
- **Time and Place:** Tuesday and Thursday 7:30pm to 9:17pm, 378 MSC.
- **Office hours:** Tuesday and Thursday 4:00pm to 5:00pm or by appointment. (You are encouraged to communicate with me by email. I check my email several times a day including weekends.)
- **Website:** *http://www.oakland.edu/∼echeng/apm6664.html*.

Objective:
Discrete optimization covers optimization problems with feasible sets containing discrete points. For example, matching job candidates with positions available is a discrete optimization problem as one cannot hire half a person. *Combinatorial optimization* is a branch of it in which we study the interface between combinatorics and optimization. The principal goal is to design good algorithms for solving special subclasses of these discrete optimization problems based on combinatorics. This is also a part of a broader subject, namely, *operations research*. Some of the materials are already discussed in APM 563. In this course, we give an in-depth treatment of these topics and study their connections with linear programming. Programming is not required for this course but it can be incorporated into the course for those who are interested in the study of efficient implementation of algorithms.

Text: *Combinatorial Optimization* by Cook, Cunningham, Pulleyblank and Schrijver.

Evaluation:
Assignments: 20% of your grade. A midterm: 30% of your grade. A final exam: 50% of your grade. I may offer you alternate evaluation scheme. (This means more choices may be available to you.)

Syllabus:
Instead of giving you a syllabus, I am attaching the topics that were covered from the last few times. See webpage for links.

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- Office Hours: Tuesday 4:00pm to 5:00pm, Thursday 4:00pm to 5:00pm and by appointment
- The authors of the text has a webpage for the book including a list of typographical errors.
- Topics covered varies from semester to semester and in part depends on the interests of the students. Take a look at what was covered in Fall 2002 to get an idea: Topics covered in Fall 2002. Take a look at what was covered in Fall 2007 to get an idea: Topics covered in Fall 2007. Take a look at what was covered in Winter 2013 to get an idea: Topics covered in Winter 2013.

Topics covered

- Jan 4: Review of LP: Fourier-Motzins Elimination, Farkas' Lemma.
- Jan 9: Review of LP: Farkas' Lemma, Strong Duality Theorem, Fundamental Theorem of Linear Programming, Complementary Slackness.
- Jan 11: Minimum weight spanning tree problem, Kruskal's algorithm, Prim's algorithm, basic polyhedral concepts.
- Jan 16: Basic polyhedral concepts, spanning tree polytope, shortest paths, feasible potentials, Ford algorithm.
- Jan 18: Shortest paths, feasible potentials, Ford algorithm, Bellman-Ford algorithm.
- Jan 23: Linear programming and shortest path, Dijkstra's Algorithm, maximum flow and minimun cut problems.
- Jan 25: Duality proof of the max-flow-min-cut theorem, complementary slackness proof of the max-flow-min-cut theorem, TU proof of the max-flow-min-cut theorem.
- Jan 30: Ford-Fulkerson Algorithm, Edmonds-Karp Modification, max-flow-min-cut theorem, Konig's Theorem.
- Feb 1: Gale's Theorem, Hoffman's Theorem, feasibility results, Menger's Theorems.
- Feb 6: Min-Flow-Max-Cut Theorem, Baseball team eliminatin, Karger's algorithm of finding unrestricted min cuts.
- Feb 8: Node identoficaton algorithm and Karger's algorithm of finding unrestricted min cuts, min cost flow problem.
- Feb 13: Min cost flow problem, perfect matching algorithm for bipartite graphs, Hall's Theorem.
- Feb 15: Maximum cardinality matching algorithm for bipartite graphs, Konig's Theorem, perfect matching algorithm for general graphs, Tutte's Theorem.
- Feb 27: Weighted perfect matching algorithm for bipartite graphs.
- Mar 1: No class (weather)
- Mar 6: TSP presented by Professor Steffy.
- Mar 8: Q&A.
- Mar 13: Weighted perfect matching algorithm for general graphs.
- Mar 15: Maximum cardinality matching algorithm for general graphs, Tutte-Berge Formula (two proofs), Edmonds-Gallai Decomposition, intergrality of LP relaxation of maximum weight matching problem of bipartite graphs.
- Mar 22: Integrality of LP relaxation of minimum cost covering problem of bipartite graphs,

polyhedral theory.

- Mar 27: Minkowski-Steinitz-Weyl-Farkas Theorem, Helly's Theorem, Radon's Theorem, Shapley-Folkman Theorem.
- Mar 29: Selected topics from 6.1-6.5.
- April 3: Selected topics from 6.1-6.5.
- April 5: TSP app.
- April 10: Selected topics from 6.1-6.5, stable set polytope.
- April 12: Stable set polytope.
- April 17: Stable set polytope.

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FALL 2002

We have covered the following in this course.

1. Linear Programming
   - Simplex method
   - Bland's Rule
   - Duality Theorem
   - Optimality Condition
   - Farkas' Lemma
2. Shortest Paths
   - Ford-Bellman-Moore algorithm
   - Feasible potentials
   - Detecting negative cycles
   - Dijkstra's algorithm
3. Minimum Weight Spanning Trees
   - Kruskal's algorithm
   - Prim's algorithm
   - Borůvka's algorithm
   - Spanning Tree Polytope
4. Maximum Flows
   - Max-Flow Min-Cut Theorem
   - Max-Flow Min-Cut Theorem via LP duality
   - Ford-Fulkerson algorithm
   - Edmonds-Karp Modification
   - Karger's algorithm

- Circulation theorems
- Applications to cardinality problems and min-max results of vertex-cover, matching, stable set, edge-cover for bipartite graphs.

5. Minimum-Cost Flows
   - Optimality Condition via linear programming
   - Optimality Condition wrt negative cycle
   - Karp's minimum mean weight negative cycle algorithm
   - Polynomial time negative cycle augmentation algorithm
6. Matching (Cardinality Version)
   - Konig's Theorem and augmenting tree algorithm for bipartite matchings
   - Tutte's Matching Theorem (two proofs, algorithmic and non-algorithmic)
   - Tutte-Berge Formula (two proofs, algorithmic and non-algorithmic)
   - Edmonds' Blossom Algorithm for matchings
   - Gallai-Edmonds Decomposition
7. Matching (Weighted Version)
   - Hungerian algorithm for the weighted bipartite matchings
   - Edmonds' weighted matching algorithm
   - Perfect Matching Polytopes
   - Minimum weight edge-cover
   - Chinese Postman Problem
8. Basic Polyhedral Theory
   - Convex sets, convex hull, polyhedra, polytopes, totally unimodular matrices and Carathéodory's Theorem
   - Other results such as Minkowski-Weyl Theorem are stated without proof.
9. Stable Set Polytopes
   - Cycle inequalities and wheel inequalities

- Separation algorithms
- Branch-and-cut

10. Approximation algorithm
    - Unless P=NP, there is no constant approximation algorithm for TSP.
    - Christofides' algorithm for metric TSP
    - Approximation algorithms via greedy heuristic and LP relaxation for knapsack problem, vertex cover problem, acyclic subgraph problem and maxcut.

11. A very very very brief introduction to matroids